SOLUTION TO FINAL EXAMINATION

Course name: MATHEMATICAL STATISTICS FOR ENGINEERS

SEMESTER 2 - ACADEMIC YEAR 2019-2020

Q1: Let $A_i = \left\{hard\ drives\ from\ plant\ i\right\}, i = 1,2,3$,

$B = \left\{a\ randomly\ selected\ hard\ drive\ is\ defective\right\}$ .25 points

a. $P\left(B\right) = \sum_{i=1}^{3} P\left(A_i\right) P\left(B \mid A_i\right) = .54 \times .04 + .35 \times .08 + .11 \times .12 = \frac{157}{2500} = .0628$ .25 points

b. $P\left(A_2 \mid B\right) = \frac{P\left(A_2\right)P\left(B \mid A_2\right)}{P\left(B\right)} = \frac{.35 \times .08}{.0628} = \frac{70}{157} \approx .4459$ .5 points

Q2:

a. $X = number\ of\ defective\ tires\ you\ find\ before\ you\ finding\ 4\ good\ tires$

$\to X \sim NegBin\left(4, .95\right)$

$$P\left(X \le 2\right) = \sum_{x=0}^{2} C_{x+4-1}^{4-1} \times .95^4 \times .05^x \approx .99777$$ .5 point

b. $E\left(X\right) = \frac{r\left(1-p\right)}{p} = \frac{4}{19}, \qquad V\left(X\right) = \frac{r\left(1-p\right)}{p^2} = \frac{80}{361}$ .5 points

Q3: Let $X = number\ of\ integrated\ circuits\left(ICs\right) is\ faulty \to X \sim H\left(25,5,4\right)$ .5 points

The probability this shipmet of 25 ICs will be accepted:

$$P\left(X \le 1\right) = \sum_{x=0}^{1} \frac{C_5^x C_{20}^{4-x}}{C_{25}^4} = \frac{2109}{2530} \approx .8336$$ .5 points

Q4: Let $X = free\ speeds\ can\ best \to X \sim N\left(118, 13.1^2\right)$, $Z = \frac{X-118}{13.1} \to Z \sim N\left(0,1\right)$ .25 points

a. $P\left(X > 100\right) = 1 - \Phi\left(-\frac{180}{131}\right) = .915286$ .25 points

b. $Y = number\ of\ vehicles\ is\ not\ exceeding\ the\ posted\ speed\ limit$

$\to Y \sim Bin\left(5, p\right)$, with $p = P(X \le 100) = .084714$ .5 points

$$P\left(Y \ge 1\right) = \sum_{x=1}^{5} C_5^x p^x \left(1-p\right)^{5-x} \approx .3576$$ .5 points

Q5:

a. $n=16, \bar{x}=301.1875, s_x=91.3668202$ .25 points

$\gamma=.95 \to \alpha=.05 \to t_{\alpha/2,n-1}=2.131$ .25 points

A 95% confidence interval for the population mean is:

$$\mu \in \bar{x} \pm t_{\alpha/2,n-1}\frac{s}{\sqrt{n}}=(252.5118265, 349.8631735)$$ .5 points

b. Let $\mu = \textit{the average amount of water is filled in 1 liter bottles}$ .25 points

Null hypothesis $H_0: \mu=1$ Alternative hypothesis $H_a: \mu<1$ .25 points

$\alpha=.01 \to z_\alpha=2.3265$

$z=(.996-1)\frac{\sqrt{20}}{.008}\approx -2.2361$ Reject H_a .25 points

That the average fill is 1 liter. .25 points

Q6:

a. $\mu_A = \textit{average worker productivity after wearing noise reduction device}$

$\mu_B = \textit{average worker productivity before wearing noise reduction device}$ .25 points

Let $\mu_d=\mu_A-\mu_B$

Null hypothesis $H_0: \mu_d=0$ Alternative hypothesis $H_a: \mu_d>0$ .25 points

we have: $n_d=15, \bar{d}=1.066666667, s_d=3.731462116$

$\alpha=.05 \to t_{\alpha,n-1}=1.761$

$t=(\bar{d}-0)\frac{\sqrt{n_d}}{s_d}\approx 1.1071<t_{\alpha,n-1} \to$ Reject H_a .5 points

That wearing the noise reduction head gear increases worker productivity is wrong. .25 points

b. $p_m = \textit{proportions of men said "yes"}, p_w = \textit{proportions of women said "yes"}$ .25 points

We have: $f_m=\frac{291}{349}, f_w=\frac{217}{336}, f=\frac{508}{685}$

Null hypothesis $H_0: p_m=p_w$ Alternative hypothesis $H_a: p_m \neq p_w$ .25 points

$\alpha=.02 \to z_{\alpha/2}\approx 2.3265$

$$z=\frac{f_m-f_w}{\sqrt{f(1-f)\left(\frac{1}{349}+\frac{1}{336}\right)}}\approx 5.6184 \notin (-z_{\alpha/2}, z_{\alpha/2}) \to$$ Reject H_0 .5 points

That different proportions of men and women in this student population would be willing to marry beneath their class. .25 points

Q7: The correlation coefficient: $r=0.8764526758$ .25 points

The equation of least-squares regression line for predicting metabolic rate from body mass: .25 points

$y=201.1615996+24.02606662x$ .25 points

When $x=45 \to y=1282.334598$ .25 points